Varianta 046

Subiectul I

a)1. b) $\sqrt{2}$. c)1. d) a = 1; b = -1. e) $S = \frac{3}{2}$. f) c = -11, b = 60.

Subiectul II

1. a) $\hat{0}$. b) 1. c) $\frac{1}{25}$. d) $x = \frac{1}{4}$. e) $\frac{3}{5}$.

2. a) $f'(x) = 5x^4 + 7$. b) $\frac{2}{3}$. c) $f'(0) = 7$. d) $f'(x) > 0, (\forall)\, x \in \mathbf{R}$. e) 1

Subiectul III

a) $(X * Y) * Z = X * (Y * Z) = XYZ + XY + XZ + YZ + X + Y + Z$.

b) $E = 0$.

c) $X = \begin{pmatrix} a_1 & b_1 \\ 0 & a_1 \end{pmatrix};\ Y = \begin{pmatrix} a_2 & b_2 \\ 0 & a_2 \end{pmatrix};\ X * Y = \begin{pmatrix} a_1a_2 + a_1 + a_2 & a_1a_2 + a_2b_1 + b_1 + b_2 \\ 0 & a_1a_2 + a_1 + a_2 \end{pmatrix}$

$a_1a_2 + a_1 + a_2 + 1 = (a_1 + 1)(a_2 + 1) \neq 0$.

d) $I'I_2 + I' + I_2 = 0 \Rightarrow I' = -\frac{1}{2} I_2$.

e) $X^2 + 2X = 3I_2;\ X = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \Rightarrow a^2 + 2a = 3, b(a+1) = 0 \Rightarrow a \in \{1, -3\}$ si $b = 0 \Rightarrow X_1 = I_2, X_2 = -3I_2$.

f) $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}^n = \left(aI_2 + \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \right)^n = a^n I_2 + C_n^1 a^{n-1} \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{pmatrix}$ (deoarece $\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}^k = 0, (\forall) k \geq 2$).

g) $n = 2,\ I_2 * I_2 \overset{(c)}{=} 3I_2$

$I_2 * I_2 * \dots * I_2 * I_2 = \left((2^n - 1)I_2\right) * I_2 = 2(2^n - 1)I_2 + I_2 = (2^{n=1} - 1)I_2$.

Subiectul IV

a) $f'(x) = 1 - \sin x \geq 0, (\forall) x \in \mathbf{R}$ si $f'(x) = 0 \Leftrightarrow x \in \{\frac{\pi}{2} + 2k\pi \mid k \in Z\}$ f este strict crescatoare

$\Rightarrow \lim_{x \to -\infty} f(x) = -\infty, \lim_{x \to \infty} f(x) = +\infty$ f este continua (cu probrietatea lui Darboux)=> Imf=**R**

b) $b_n = f^{-1}(n)$, f^{-1} este strict crescătoare deci $b_{n+1} > b_n, n \in \mathbf{N}$.

c) Deoarece $(b_n)_n$ este monoton există $\lim_{x \to \infty} b_n = \lim_{x \to \infty} f^{-1}(n) = \lim_{x \to \infty} f^{-1}(x)$.

Dacă $\lim_{x \to \infty} b_n = b \in \mathbf{R}$, atunci $\lim_{x \to \infty} f^{-1}(x) = b \Rightarrow f(b) = \lim_{x \to \infty} f(f^{-1})(x)) = \infty \Rightarrow \lim_{x \to \infty} b_n = \infty$.

d) $b_n + \cos b_n = n \Rightarrow \frac{b_n}{n} + \frac{\cos b_n}{n} = 1 \Rightarrow \lim_{n \to \infty} \frac{b_n}{n} + 0 = 1$

e) Daca $x \in (0, \frac{\pi}{2})$, atunci din a=> $f(x) \in (f(0), f(\frac{\pi}{2})) = (1, \frac{\pi}{2}) \subset (o, \frac{\pi}{2})$.

Daca $a_0 \in (0, \frac{\pi}{2}) \Rightarrow a_n \in (0, \frac{\pi}{2}), n \in N$. $a_{n+1} - a_n = \cos a_n > 0$.

f) Daca

$x \in (\frac{\pi}{2};\pi)$, atunci din a) => $f(x) \in (f(\frac{\pi}{2}), f(\pi)) = (\frac{\pi}{2}, \pi - 1) \subset (\frac{\pi}{2}, \pi)$,

deci $a_0 \in (\frac{\pi}{2};\pi) \Rightarrow a_n \in (\frac{\pi}{2},\pi), n \in \mathbf{N}$

$a_{n+1} - a_n = \cos a_n < 0.$

g) Din e) si f) si din $a_0 = \frac{\pi}{2} \Rightarrow a_n = \frac{\pi}{2}, n \in$ N, rezulta ca sirul $(a_n)_n$ este monoton si marginit pentru orice $a_0 \in (0,\pi)$. Deoarece f este continua $a = \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n) \Rightarrow$

$a = f(a) \Leftrightarrow a + \cos a = a, a \in [0,\frac{\pi}{2}] \Rightarrow a = \frac{\pi}{2}.$